CRUSTAL STRUCTURE FROM WAVEFORM INVERSTION OF SHEAR-COUPLED PL

Jay Pulliam, Mrinal K. Sen, Cliff Frohlich, and Steve Grand

The University of Texas at Austin

Sponsored by the Defense Threat Reduction Agency

Contract No. DTRA01-00-C-0060

ABSTRACT

One strategy for discriminating between explosions and natural events depends on accurate determinations of event locations, including focal depths. If a seismic event could be reliably determined to have a focal depth greater than a few kilometers, one could be confident that the event is not an explosion. But to determine focal depths accurately, one must first have a fairly accurate model of the crustal structure in the vicinity of the event. Unfortunately, sufficiently accurate models do not exist for many regions of interest to the nuclear explosion monitoring community. Our previous work focused on developing and evaluating strategies for locating events using a single three-component seismic station; velocity models were obtained via crustal receiver function modeling, and waveform correlation methods were used to determine focal depths for which synthetics fit the data best. However, for an event at a regional distance from a given station, the sampling provided by the teleseismic phases used for receiver functions is not ideal. These waves tend to approach the station at a steep angle, sampling just a narrow cone beneath the station.

Better sampling is provided by shear-coupled PL (SPL) phases, which sample the crust over 1000 km or more as they approach the station. This sampling provides a better lateral average of the crust and more closely resembles the sampling of phases emanating from seismic events at regional distances. Our current research centers on modeling SPL phases using a novel modeling algorithm that uses the reflectivity method to compute synthetic seismograms while holding deeper portions of the mantle fixed, in terms of pre-computed and stored reflectivity and transmission matrices. Layers of the crust and upper mantle are allowed to vary over broad ranges and the entire algorithm is powered by a variant of simulated annealing, a global optimization method. Using the results of all trials we are able to compute the posterior probability distribution for model parameters and thereby assess the strength of the constraints placed upon the model parameters by the data.

We report on progress in developing the modeling code and demonstrate, through synthetics computed for receiver function models produced for China, that the sampling provided by SPL is significantly different from that provided by receiver functions. Mismatches between synthetics and data for S-wave windows including the Sp converted phase, SsPmP, and SPL, indicate that each model, while it may represent structure in the immediate vicinity of a station, does not reflect structure over a broader area around the station. Given the sampling of the SPL phase, trial-and-error modeling is very difficult and time-consuming. These results confirm, however, that Sp, SsPmP and SPL phases carry additional, valuable information that can help constrain crustal and upper mantle velocity models.

KEY WORDS: discrimination, event location, crustal structure, wave propagation

OBJECTIVE

Introduction

"Receiver function" methods, which typically use the reverberations of P phases, contained in the P coda, to constrain a 1-D model beneath a seismographic station, have been quite successful in matching important characteristics of existing refraction models. Receiver function methods allow the crust beneath a given station to be estimated with much less effort and cost than are required for active-source experiments. However, because receiver function methods use rays that arrive at the station with steep

angles of incidence (e.g., Zhao and Frohlich, 1996), they sample only a narrow cone beneath the station. The result is a 1-D model that does not represent a broad regional average, i.e., the medium through which signals from a later event of unknown provenance and location would propagate. Furthermore, the predominantly vertically propagating reverberations that make up the P coda reflect steeply off the crust's internal layers. They therefore do not emulate the propagation of regional phases, which reflect at more oblique angles or are refracted by these layers. This is a problem for locating and/or determining focal depths of small, regional seismic events. In short, because of the data they use, the models produced by receiver function methods may be inadequate for the purposes of Comprehensive Nuclear-Test-Ban Treaty (CTBT) monitoring.

We are developing a new method for modeling the crust and upper mantle that retains the time and cost advantages of P-coda receiver function methods but which uses types of data that are more appropriate for CTBT purposes: Shear-coupled PL phases (SPL), Sp phases converted at the Moho, and SsPmP. SPL samples the crust and upper mantle in the vicinity of a station most broadly compared to Sp, SsPmP and P. Modeled simultaneously (where they exist), SPL, Sp, and SsPmP offer the potential for producing azimuthally dependent structural models.

What is shear-coupled PL? Frazer (1977) and Baag and Langston (1985) review previous work on SPL extensively; we will summarize aspects that are relevant for our proposed CTBT monitoring research.

The conventional S phase is the initial, relatively sharp and pulse-like arrival that signals the beginning of a wavetrain with generally longer periods and normal dispersion. The particle motion associated with the S phase is rectilinear, and all three components of motion are in phase. The dispersive wavetrain that follows S exhibits prograde elliptical particle motion that is confined to the vertical plane. Oliver (1961) named this wavetrain "shear-coupled PL" because it is analogous to the PL wavetrain, which appears between P and S arrivals at regional distances. Oliver (1961) presented a theory, based on the observed group and phase velocity of SPL that explained the phase as coupling between S and the fundamental leaking mode of Rayleigh waves in the crustal waveguide. According to this theory, shear energy radiated by an earthquake (or explosion) travels through the Earth as a body phase, whereupon it impinges upon the Moho. Afterward it travels through the crustal waveguide as trapped P-waves and leaky SV-waves. The only difference between a PL phase, which is observed at regional distances from a source, and SPL phases, which are observed at teleseismic distances, is that SPL is generated by a shear wave impinging upon the Moho at regional distances from the observing station. Chander et al. (1968), Frazer (1977), and Baag and Langston (1985) essentially validated Oliver's hypothesis by presenting computational methods for synthesizing SPL. However, Chander et al. (1968) present only approximate seismograms and do not include S waves. Frazer's method does not include the effects of anelastic attentuation, nor did he explore the effects of near-source structure on SPL generation and propagation.

While previous work has elucidated the origin and propagation characteristics of SPL (e.g., Oliver, 1961; Chander et al., 1968; Alsop and Chander, 1968; Poupinet and Wright, 1972; Frazer, 1977; Baag and Langston, 1985) attempts to use a portion of the S coda to model the crust explicitly are quite limited (e.g., Helmberger and Engen, 1974; Swenson et al., 1999). Modeling of Sp and SsPmP have received slightly more attention (e.g., Jordan and Frazer, 1975; Langston, 1996). This disparity is understandable; synthesizing SPL is computationally intensive (Frazer, 1977; Baag and Langston, 1985). Because of the difficulty in computing waveforms, the inverse problem of determining structure along the propagation path from observed SPL has not been adequately explored. Our objectives are (a) to evaluate the usefulness of shear-coupled PL, Sp, and SsPmP phases for modeling crustal and upper mantle structure using real and synthetic data, (b) develop a waveform inversion technique based on a novel implementation of the reflectivity method and global optimization algorithms, and (c) apply the inversion technique to model the crust and upper mantle beneath China. Figure 1 shows examples of synthetic seismograms computed for three models that differ in their upper mantles and crusts. One can clearly see the sensitivity in timing, pulse width, and relative amplitudes of S, Sp, SsPmP, and SPL phases. Successful modeling of data that contain these phases will provide strong constraints on regional P and S velocity models.

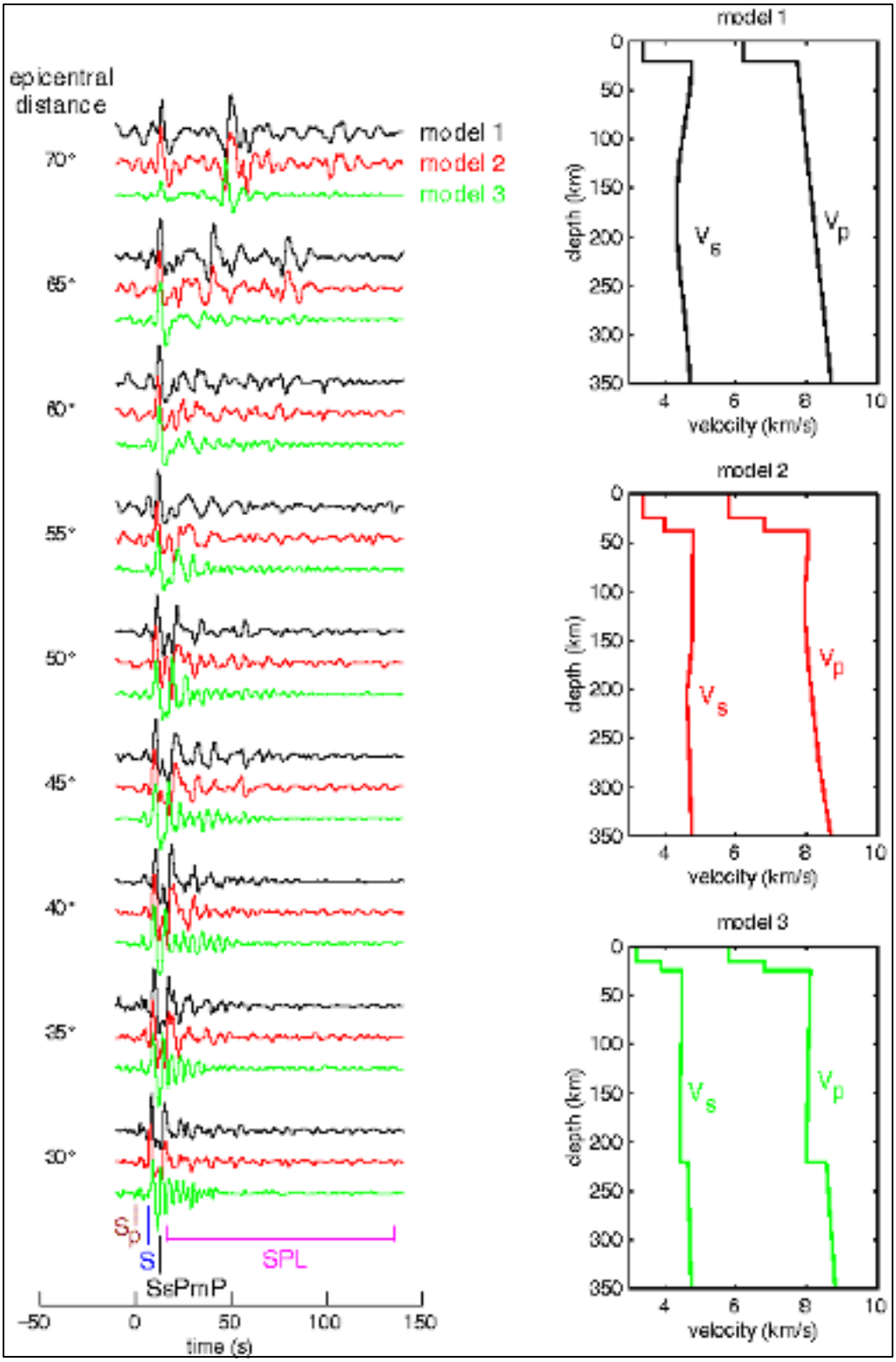

Figure 1. A comparison of vertical-component synthetic seismograms showing S, Sp, SsPmP, and SPL waves computed using the models shown at right. These seismograms were computed using our reflectivity code for frequencies between 0.001 and 1 Hz and a dip-slip dislocation source at 600 km depth.

RESEARCH ACCOMPLISHED

Development of the Modeling Technique

The reflectivity calculation involves computation of reflectivity matrices for a stack of layers as a function of ray parameter (or wavenumber) and frequency. The computation of reflectivity responses for different ray parameters and frequencies is completely independent of each other. We took advantage of this independence to develop a reflectivity code that can be run on parallel computer architectures. We parallelized our code over the ray parameters, i.e., to each node we assign a certain number of ray parameters to compute. The number of ray parameters to be computed can be determined based on the processor speed. If all the processors are of the same speed, they are assigned equal number of ray parameters. At the end, the master node assembles the partial responses and performs the inverse transformation to generate synthetic seismograms at the required azimuths and distances.

We used MPI for message passing and ran our code on a PC cluster consisting of 16 nodes; each node is a 660-MHz alpha processor with 8-MB cache and 1-GB of RAM. A Myrinet interconnect is used to communicate between nodes. Figure 2 shows timing statistics for the reflectivity code on our PC cluster.

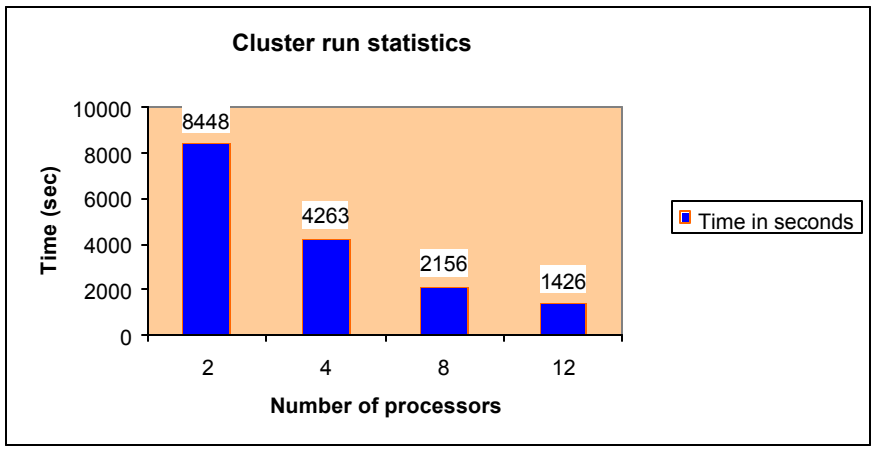

Figure 2. The statistics of an example run to compute complete seismograms for frequencies from 0 to 0.5 Hz (number of frequencies = 2000, no of layers=192, no of ray-parameters=6000) on the DEC Alpha cluster. Note the near linear speedup of the algorithm with the increase in the number of processors.

We plan to obtain further increases in computational speed by using additional processors (we have been unable to use all 16 processors due to minor hardware difficulties), tailoring the number of ray parameters more carefully to the structure being modeled, and by pre-computing and storing reflectivity matrices for parts of the mantle that will not vary in our modeling. The high frequencies needed to compute body waves accurately over the distances to which SPL is observed contribute to the difficulty in computing SPL. We propose to essentially remove these complications by storing pre-computed products for fixed parts of the Earth model.

The modeling process will be controlled by a global optimization algorithm called Very Fast Simulated Annealing (VFSA) (e.g., Sen and Stoffa, 1995). Simulated annealing (SA) is analogous to the natural process of crystal annealing when a liquid gradually cools to a solid state. The SA technique starts with an initial model $\mathbf{m}_0$, with associated error or energy $E(\mathbf{m}_0)$. It draws a new model $\mathbf{m}_{new}$ from a flat distribution of models within the predefined limits. The associated energy $E(\mathbf{m}_{new})$ is then computed and compared against $E(\mathbf{m}_0)$. If the energy of the new state is less than the energy of the initial state, the new model is

accepted and it replaces the initial model. However, if the energy of the new state is higher than the initial state, $\mathbf{m}_{new}$ is accepted with the probability of $\exp\left(\left(E(\mathbf{m}_{new}) - E(\mathbf{m}_0)\right)/T\right)$, where T is a control parameter called temperature. This rule of probabilistic acceptance (also called the Metropolis rule) allows SA to escape local minima. The process of model generation and acceptance is repeated a large number of times with the annealing temperature gradually decreasing according to a predefined cooling schedule. A variant of SA, called Very Fast Simulated Annealing (VFSA) speeds up the annealing process by drawing each new model from a temperature dependent Cauchy-like distribution centered on the current model. This change with respect to SA has two fundamental effects. First, it allows for larger sampling of the model space at the early stages of the inversion (when "temperature" is high), and much narrower sampling in the model space as the inversion converges and the temperature decreases. Second, each model parameter can have its own cooling schedule and model-space sampling scheme. VFSA therefore allows for individual control of each parameter and the incorporation of a priori information.

Uncertainty Estimation: PPD, Posterior Covariance and Correlation

In seismic waveform inversion, more than one model can often explain the observed data equally well and trade-off between different model parameters is also common. It is therefore important not only to find a single, best-fitting solution but also to find the uncertainty and level of uniqueness of that solution. A convenient way to address this issue is to cast the inverse problem in a Bayesian framework (e.g., Tarantola, 1982; Sen and Stoffa, 1995) in which the posterior probability density function (PPD) is the answer to the inverse problem. The PPD is proportional to the product of a likelihood function and the prior. It can be shown (e.g., Tarantola, 1982) that the PPD $\sigma(\mathbf{m}|\mathbf{d}_{obs})$ is given by

$$\sigma(\mathbf{m}|\mathbf{d}_{obs}) \propto \exp(-E(\mathbf{m})). \tag{6}$$

Since the PPD is a multi-dimensional function, one generally evaluates marginal PPD, mean, covariance and some higher order moments. All these quantities can be represented by the following multi-dimensional integral

$$I = \int d\mathbf{m}\, f(\mathbf{m})\sigma(\mathbf{m}|\mathbf{d}_{obs}). \tag{7}$$

Thus, we are now left with the problem of finding efficient methods of evaluating a multi-dimensional integral. Importance sampling based on a Gibbs sampler or a Metropolis rule (Sen and Stoffa, 1996) can be used effectively to evaluate these integrals and to estimate PPD, posterior mean, covariance and correlation matrices. The posterior covariance and correlation matrices clearly demonstrate the trade-off between different model parameters. Sen and Stoffa (1995) showed that multiple VFSA runs with different random starting models could be used to sample models from the most significant parts of the model space. These models, when used to evaluate equation (7) result in estimates that are fairly close to the values obtained by theoretically correct Gibbs sampling. Multiple VFSA, however, is computationally very efficient.

Application to Data

After exploring available data and relevant previous studies, we decided to focus first on China (Figure 3), for which several studies of crustal receiver functions have been made. The fairly thick crust beneath China produces good separation between Sp, S, and SsPmP phases, which makes China an ideal location for a first application of the modeling method. We selected eight deep events at epicentral distances ranging from 22° to 58°, rotated, filtered and windowed the data around the S arrival. Figure 4 shows data for a single station (BJT, although it is the former station BJI for the 1990 event), including the results of a polarization filter (radial component multiplied by the vertical component), which aids in identifying phases. Since radial and vertical motion are correlated for rectilinear motion, P-type energy shows up as positive values whereas SV-type energy is negative. The best examples of Sp, SV, SsPmP, and SPL are marked with arrows in Figure 4. These arrivals, which also appear clearly in our sample synthetics (e.g., Figure 1), are highly sensitive to the structure of the crust and upper mantle and offer strong modeling

constraints on that structure. However, due to the nature of their propagation, modeling these phases by trial-and-error would be extremely time-consuming and frustrating—if it is feasible at all.

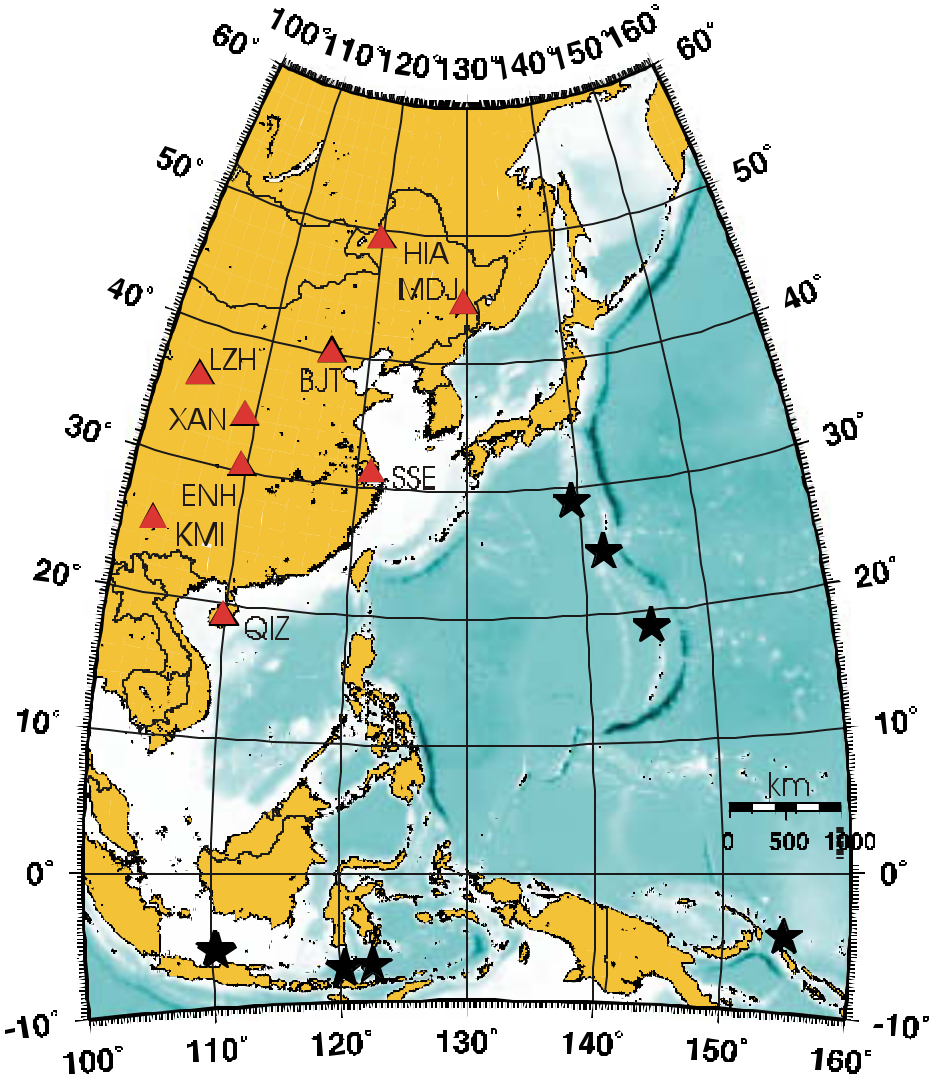

Figure 3. Map of China, including the earthquakes (stars) and stations (triangles) used in this study. See Epicentral distances range from 22° to 58°. See Table 1 for earthquake parameters.

Table 1. Events used in this study.

	Year	Month	Day	Hour	Min	Sec	Latitude (°)	Longitude (°)	Depth (km)	Mag
1	1990	5	24	20	9	23.2	-7.363	120.363	588.9	6.4
2	1994	9	28	16	39	51.7	-5.786	110.352	637.5	6.6
3	1994	11	15	20	18	11.3	-5.589	110.186	560.6	6.5
4	1995	8	23	7	6	2.8	18.856	145.218	594.9	7.1
5	1996	5	2	13	34	29.0	-4.548	154.833	500.0	6.6
6	1996	6	17	11	22	18.5	-7.137	122.589	587.3	7.9
7	1998	2	7	1	18	59.5	24.821	141.746	525.3	6.4
8	1998	8	20	6	40	55.8	28.932	139.329	440.5	7.1

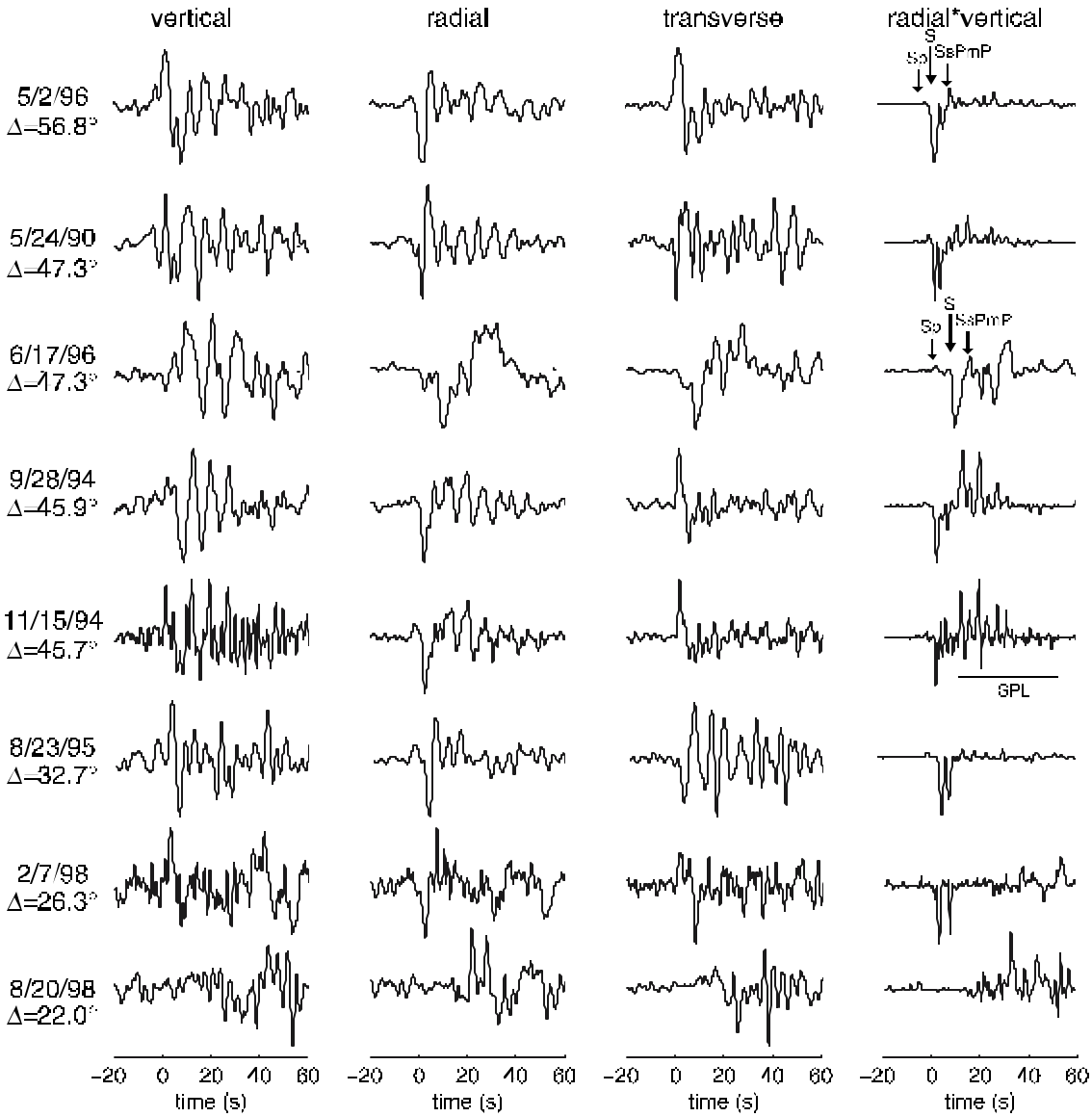

Figure 4. Vertical-, radial-, and transverse-component data from the events listed in Table 1 for the single station BJT (BJI for the 1990 event). These velocity records have been bandpass-filtered with a 5-pole Butterworth filter with corner frequencies at 0.01 and 0.5 Hz. At far right is the result of a simple polarization filter consisting of the vertical component multiplied by the radial component. Rectilinear P-type motion is indicated by positive values and rectilinear SV motion is indicated by negative values, which aids in the identification of phases.

We implemented a number of the crustal models obtained by Mangino et al. (1999) and computed synthetic seismograms via reflectivity for deep events at distances of 30°-60° to compare to the data. These P-velocity models were converted to S models by means of a simple proportionality (1.8). Synthetics were computed up to frequencies of 0.5 Hz and were convolved with the Harvard CMT solutions for each event. Our synthetics matched some aspects of the data—generally we obtained a good match to the SV wave but

failed to match the Sp, SsPmP, and SPL phases (Figure 5). Even this level of agreement is impressive. Receiver functions, as discussed above, model reverberating waves that sample the crust differently than do the shear wave converted phases. Second, the S model we used was converted simply from P models. Lastly, in order to compute synthetics over teleseismic distances, we took the liberty of inserting Mangino et al.'s models into PREM, essentially replacing the PREM crust. The structure of the uppermost mantle, including the thickness of the crust and the velocity contrast across the Moho, play crucial roles in the Sp/SV time separation, waveshapes and amplitude ratios. Modeling these aspects accurately will have to await the completion of our modeling scheme.

CONCLUSIONS AND RECOMMENDATIONS

Although we believe the method we are developing holds strong promise, we have several concerns to address. Among the issues that must be settled are the questions:

Is SPL responsible for the S coda?

An important assumption underlying this research is that at teleseismic distances, SPL waves are responsible for a significant proportion of the energy in the S coda. Our modeling approach would be invalid if the coda structure were controlled instead by fundamentally different processes, such as multipathing or scattering from lateral heterogeneity. However, we believe that the latter case is unlikely because of the considerable previous success of investigators such as Frazer (1977) and Baag and Langston (1985), who generated SPL waves to reproduce the essential features of signal that followed the S phase. Moreover, SPL generally contains considerable power in a relatively narrow frequency range, and even in the presence of scattered energy it is likely that usable signals will emerge with suitable filtering. Finally, multipathing may be responsible for coda structure at some stations. However, it is unlikely that this is true at the majority of stations; identifying the near-station geological conditions which imply that SPL does not control coda structure is an implicit goal of this project.

Do lateral variations in structure prohibit 1-D modeling via reflectivity?

We believe that the relatively long-period nature of SPL and its broad averaging properties offer an opportunity to obtain an accurate 1-D average of even laterally heterogeneous regions. The extent to which SPL's averaging holds for strong and/or short-wavelength lateral variations is unclear, but it is likely that large regions of the world will admit 1-D reflectivity modeling of SPL waves. Our method would provide an accurate (i.e., laterally-averaged) 1-D model of the propagation path, which is more practical for monitoring purposes than a 3-D model. Our contention, which we will evaluate in the course of this research, is that a 1-D model may be "good enough" to locate seismic sources in much of the Earth, provided it accurately reflects the crustal and upper mantle structure *between* the source and the station, not just *beneath* the station.

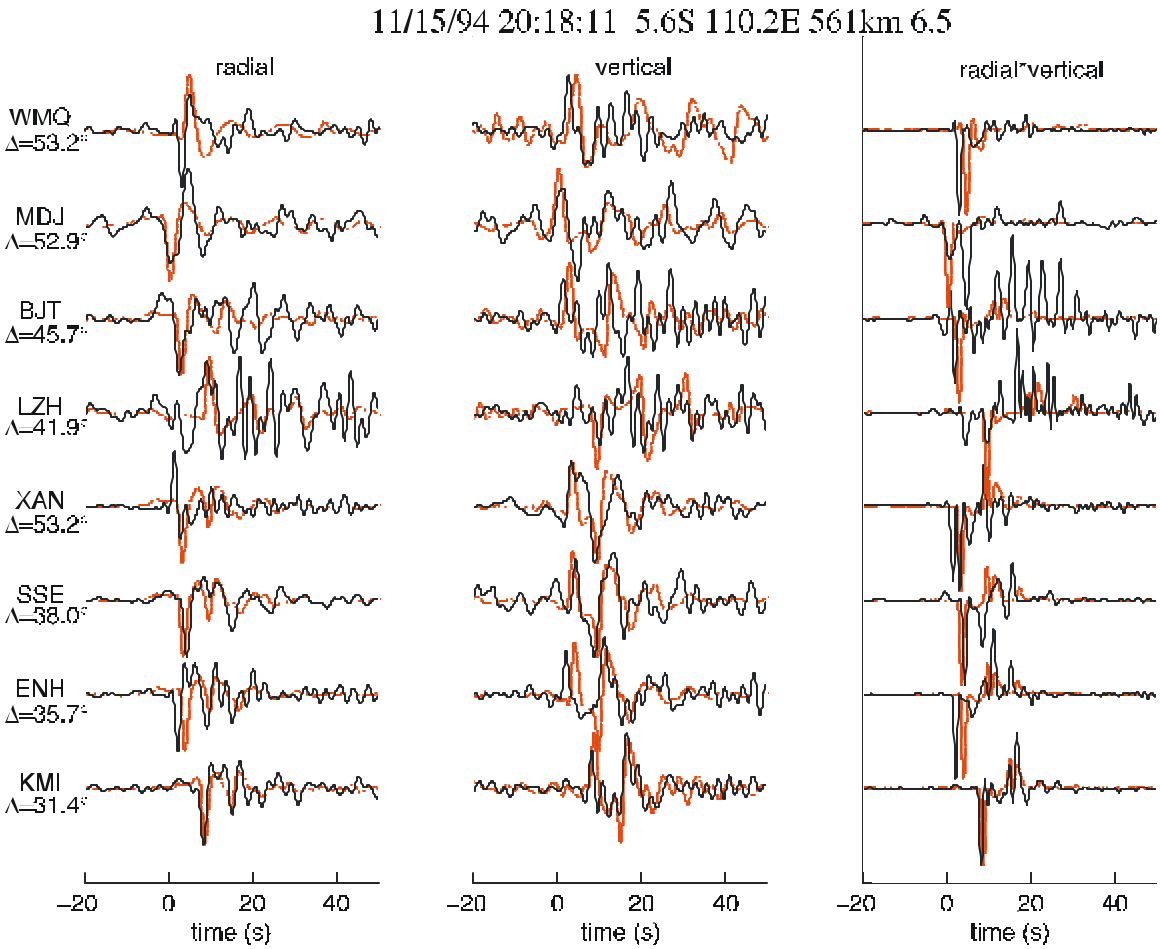

Figure 5. Comparison of data from the 11/15/94 Indonesian event (black lines) with synthetics (gray lines) generated using Mangino et al.'s (1999) crustal receiver functions inserted on top of a PREM mantle. Models were not provided for ENH, SSE, and XAN; here we show the best-fitting synthetics from among the models provided. Note that the SV arrival is well matched, in many cases, but the Sp, SsPmP, and SPL phases are generally poorly matched. The good SV match is impressive, particularly since the receiver functions consist of P velocity models—we obtain S velocities through a constant proportionality (1.8). The poor match of converted phases is not surprising, since these phases sample the crust more broadly than do the P phases used for receiver functions and therefore experience different structure. This comparison illustrates the potential usefulness of these phases for modeling regional structures in a manner most useful for monitoring purposes.

Can we avoid significant source-side contributions to SPL?

Baag and Langston (1985) elegantly demonstrate the large contribution to recorded SPL that arises from reverberations near the source for shallow earthquakes. We intend to model SPL propagation in the receiver region, which means we need to isolate receiver-side from source-side contributions. We can do this most effectively by restricting our applications to deep events. Consequently, we will have fewer events at our disposal and, ultimately, less complete coverage of the world than if we were to use both shallow and deep events. Applications to shallower events, in order obtain more complete worldwide coverage, can be explored later.

Do we have the computing power to invert SPL for velocity structure?

The process of inverting SPL to find crustal structure is highly nonlinear. Even though we will use state-of-the-art forward modeling and inverse methods, they will still require that we calculate SPL synthetics for hundreds or thousands of crustal models. Figure 2 demonstrates that we have the capacity to evaluate hundreds of models in a day's time and that computational speed increases nearly linearly with an increase in the number of processors. More and faster processors are easily obtainable, so computational constraints are not a problem.

The method we are developing will be especially useful for CTBT purposes because it will allow us to produce structural models in regions that (a) have little or no seismicity, (b) average over a broad region in the vicinity of the seismographic station, and (c) are especially well suited to modeling regional phases, due to the propagation characteristics of these converted phases. Models with these characteristics will be useful for locating small-magnitude events at regional distances with a sparse network of broadband stations, such as the International Monitoring System.

REFERENCES

Baag, C.-E. and C.A. Langston (1985), Shear-coupled PL, *Geophys. J. R. Astr. Soc.,* **80**, 363-385.

Chander, R., L E. Alsop, and J. Oliver (1968), On the synthesis of shear coupled PL waves, *Bull. Seism. Soc. Am.,* **58**, 1849-1877, 1968.

Crotwell, H.P., T.J. Owens, and J. Ritsema (1999), The TauP toolkit; flexible seismic travel-time and ray-path utilities, Seism. Res. Lett., **70**, 154-160.

Dziewonski, A. M. and D. L. Anderson (1981), Preliminary reference Earth model, *Phys. Earth. Planet. Int.,* **25**, p. 297-356.

Frazer, L.N. (1977), Synthesis of shear coupled PL. *Ph.D. thesis*, Princeton University, 54 pp.

Helmberger, D.V. and G.R. Engen (19740, Upper mantle shear structure, *J. Geophys. Res.*, **79**, 4017-4028.

Jordan, T.H. and L.N. Frazer (1975), Crustal and upper mantle structure from Sp phases, J. Geophys. Res., **80**, 1504-1518.

Langston, C.A. (1996), The SsPmP phase in regional wave propagation, *Bull. Seism. Soc. Am*., **86**, 133-143.

Mallick, S. and L.N. Frazer (1987), Practical aspects of reflectivity modeling, *Geophysics*, **52**, 1355-1364.

Oliver, J. (1961), On the long period character of shear waves, *Bull. Seism. Soc. Am.*, **51**, 1-12.

Poupinet, G. and C. Wright (1972), The generation and properties of shear-coupled PL waves, *Bull. Seism. Soc. Am.,* **62**, 1699-1710.

Sen, M. and P. L. Stoffa (1995), *Global Optimization Methods in Geophysical Inversion*, Elsevier Science Publishing Company, The Netherlands.

Swenson, J.L., S. L. Beck, G. Zandt (1999), Regional distance shear-coupled PL propagation within the northern Altiplano, Central Andes, *Geophys. J. Int.,* **139**, p. 743-753.

Tarantola, A. (1987), *Inverse Problem Theory: Methods for Data Fitting and Model Parameter Estimation*, 613 pp., Elsevier, Amsterdam.

Zhao, L.-S. and C. Frohlich (1996), Teleseismic body waveforms and receiver structures beneath seismic stations, *Geophys. J. Int.,* **124**, 525-540.