IMPROVED SEISMIC EVENT DEPTH ESTIMATION USING CEPSTRAL ANALYSIS

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ABSTRACT

The objective of cepstral analysis is to detect echoes in a signal using simple signal processing methods. In principle, cepstral analysis should be capable of detecting the delay times of seismic depth phases, which could then be used to accurately estimate event depth. Cepstral techniques have been utilized in the past for seismic depth determination with some success (Alexander, 1996; Bennett et al., 1989; Kemerait, 1982). However, the results have been mixed, at best, and the operational capability of the method has never been established. To address some of the negative aspects of the method, we have formulated a cepstral F-statistic (Shumway et al., 1998) that attaches statistical significance to peaks in the cepstrum through a signal-to-noise ratio computed from the beam cepstrum and the sum-stacked cepstrum. We estimate the cepstra from windows of data, which contain the P arrival and its coda. The F-statistic is then formed by dividing the beam (mean) cepstrum by the error between the beam and the sum-stacked cepstra. Peaks at possible depth-phase delay times must appear in both the beam cepstrum and the F-statistic estimate. Appropriate F-statistics for both arrays and single 3-component stations have been developed.

Initial results indicate that out-of-band noise in the signal severely degrades the estimate of the depth phase delay times. To address this problem, we are experimenting with both tapering and filtering ('liftering' in the cepstral literature) the detrended log spectrum of each window of data prior to forming the cepstrum. The improvement in peak detection capability from the cepstral F-statistic is significant using even a simple taper on the detrended log spectrum, and we anticipate that more sophisticated filtering methods will further improve the results. Results are presented from events from regions of monitoring concern, with an emphasis on events that have teleseismic depth phases, but confusing regional signals.

Key Words: seismic focal depth, cepstral analysis, depth-phase delay times, F-statistic

OBJECTIVE

The primary objective of this research is to establish the operational usefulness of cepstral techniques to determine the depth of a seismic event. To accomplish this objective, we have developed a statistical criterion to accompany cepstral estimates of seismic depth phase delay times. This statistical measure, or cepstral F-statistic, provides a measure of the significance of a particular peak in the cepstrum. We have applied the technique to both synthetic and Reviewed Event Bulletin (REB) data with initial results that are promising.

RESEARCH ACCOMPLISHED

Accurate determination of the focal depth of seismic events continues to be an important seismic monitoring problem. If the depth of an event can be confidently found to be more than a few kilometers beneath the surface, further event processing becomes unnecessary. However, robust techniques for determining depth from both teleseismic and regional data continue to be elusive. Depth phase signals (pP and sP) are the best indicators of depth, but they can be difficult to detect in seismic data due to path effects, focal mechanism and coda complexity. Some of the currently available techniques to determine the depth of an event from available depth phase data include body-wave modeling (Goldstein and Dodge, 1998), beamforming (Woodgold, 1999), and cepstral techniques (Alexander, 1995). In our project we are investigating a modified cepstral technique to conclusively determine its operational capability to estimate focal depths.

Cepstral techniques are used to detect echoes in a signal using simple Fourier transform processing methods. They have been used in such diverse applications as speech analysis, image processing and radar data analysis (Bogert et al., 1963; Childers et al., 1977). The cepstrum is defined as the Fourier transform of the log of the spectrum of a time domain signal (Oppenheim and Schafer, 1975), and was designed to help analyze the periodicity that occurs in the power spectrum when echoes are present in the original signal. In principle, cepstral analysis can be used to detect the periodicity of scalloping in the seismic spectrum that is due to the interference of the direct (P) and depth (pP and/or sP) phases. This periodicity is directly related to the depth phase delay time, which is in turn directly related to focal depth. Cepstral techniques have been utilized in the past for seismic depth determination (Alexander, 1996; Bennett et al., 1989; Kemerait, 1982), but the results have been mixed, and the operational capability of the method has never been established. To improve the method, we have formulated a cepstral F-statistic (Shumway et al., 1998) that attaches statistical significance to peaks in the cepstrum. The F-statistic is found using a classical approach to detecting a signal in N stationarily correlated time series (Shumway, 1971). Testing the hypothesis $S(d) = 0$, where S is the signal transform at delay time d, leads to an F-statistic given by

$$F_{2,2(N-1)}(d) = (N-1)\frac{SCB(d)}{SCE(d)},$$

which can be interpreted as a signal-to-noise ratio. The subscripts refer to an F distribution with 2 and 2(N-1) degrees of freedom. In this instance, the signal consists of the spectrum of the stacked log spectra, or *beam cepstrum (SCB),* defined as

$$SCB(d) = \sum_{j=1}^{N} N\left|\overline{Q}(d)\right|^2,$$

where

$$\overline{Q}(d) = N^{-1}\sum_{j=1}^{N} Q_j(d),$$

is the mean Fourier transform of the log spectra. The noise is defined as the *error cepstrum (SCE):*

$$SCE(d) = \sum_{j=1}^{N}\left|Q_j(d) - \overline{Q}(d)\right|^2$$

$$= SCT(d) - SCB(d),$$

where *SCT(d)* is the *total stacked cepstrum*,

$$SCT(d) = \sum_{j=1}^{N} \left| Q_j(d) \right|^2 .$$

The error cepstrum is a measure of the extent to which the individual channel or window transforms differ from the mean transform. The beam and total cepstra are computed from multiple windows of data, which can be extracted from elements in an array or from sliding windows in a single seismogram. Note that the total stacked cepstrum (*SCT(d)*) is equivalent to the sum-stack proposed by Alexander et al. (1995), which is computed by simply adding up the cepstra of individual windows. Our tests have shown that the sum-stack will not reflect the common signal components as well as either the beam cepstrum or the F-statistic.

In practice the individual cepstra require some additional processing in order for peaks that correspond to the depth phases to be detected. Figure 1 illustrates a simple flow chart of the algorithm. The log-power spectrum is calculated from each window of data. The log spectrum is then detrended with a cubic spline to remove the Fourier component of the log-power spectrum that is not due to the depth-phase echo. In the standard version of cepstral peak detection, each individual detrended log spectrum is used to form the beam and total cepstrum necessary for peak detection with the F-statistic. However, we have found that noise in the high frequency portion of the detrended log spectrum obscures peaks in the cepstrum that are due to true signal echoes. To illustrate this effect,

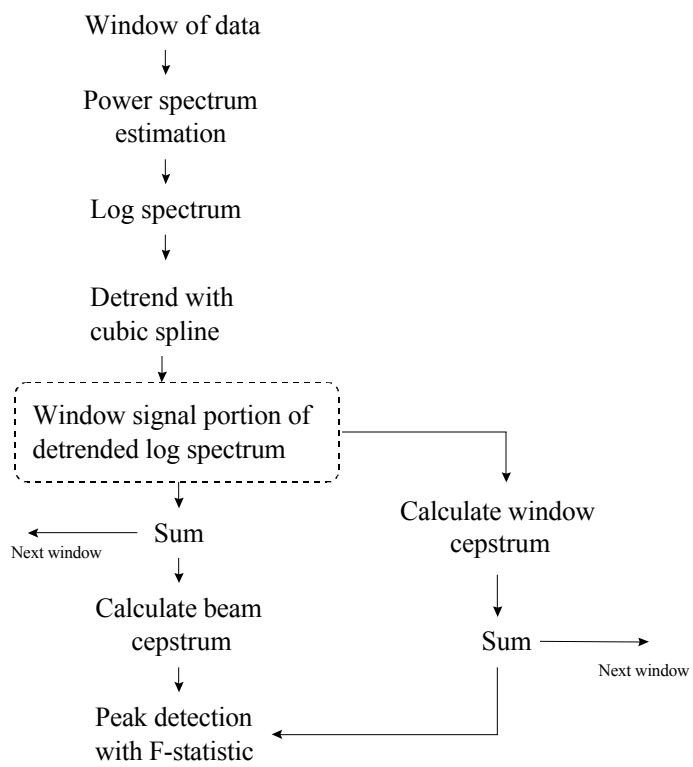

Figure 1. Flow chart illustrating the cepstral F-statistic algorithm. The solid, squared-off boxes denote steps in the algorithm that are traditional in cepstral analysis techniques. The traditional method is highly sensitive to noise in the seismic signal, which is magnified in the detrended log spectrum. Therefore, we have added a modification denoted by the dashed, rounded box, in which we window the signal portion of the detrended log spectrum prior to cepstrum estimation and peak detection.

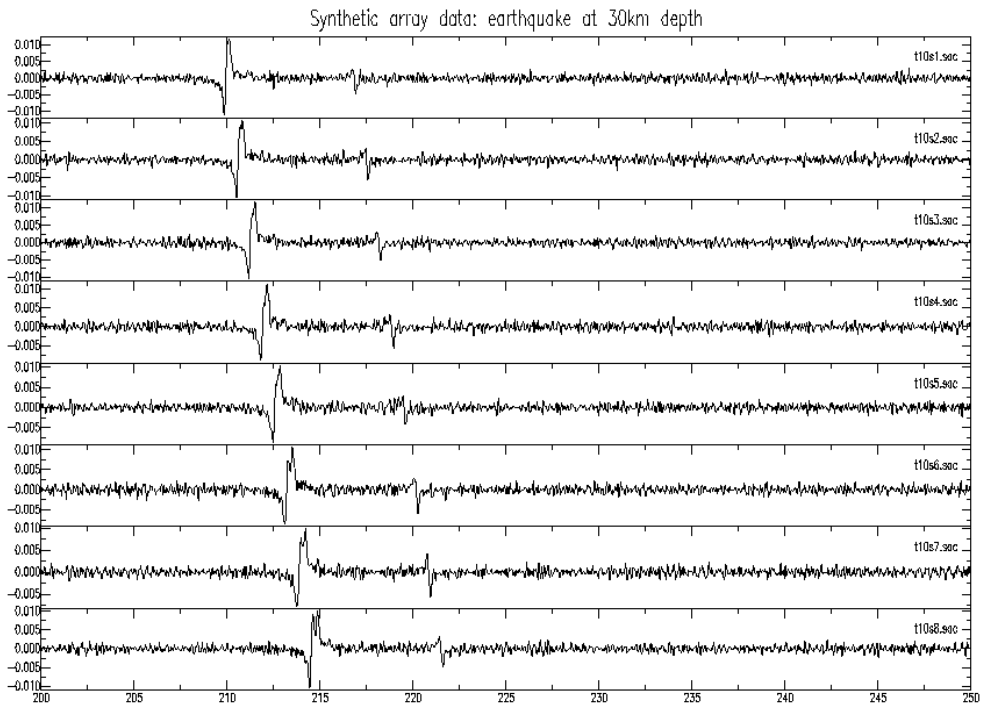

Figure 2. Synthetic data calculated using WKBJ theory. The earthquake was modeled at a depth of 30 km through the IASPEI91 velocity model. An array of eight linearly configured stations was positioned starting 15° from the epicenter with a total aperture of 39 km. Bandlimited Gaussian noise was added at a signal to noise (SNR) ratio of 10.

we modeled some noisy synthetic data using a WKBJ raytracing package (Chapman et al., 1991); the resulting seismograms are shown in Figure 2. An earthquake depth of 30 km depth and the IASPEI91 model (Kennett and Engdahl, 1991) were used to synthesize just the P and pP phase arrivals. We placed an array of eight linearly configured stations starting 15° from the epicenter, with a total aperture of approximately 39 km. After producing the seismograms, we added bandlimited Gaussian noise to the synthetic data at a signal-to-noise ratio (SNR) of 10. The delay time between the P and pP phases is approximately 6.8 seconds. We then processed the data using the cepstral technique illustrated in Figure 1, but left out the step outlined by the dashed line. The results are shown in Figure 3, where we display the beam cepstrum and total cepstrum in the top two panels. We have included the total cepstrum in the results to illustrate the superiority of the beam cepstrum and F-statistic for peak detection. The cepstral F-statistic, shown in the bottom panel, provides a statistical level of significance for the various peaks. At a false alarm rate of .05 (dashed line), multiple peaks are significant, even though only one of them corresponds to the true depth phase delay time. At a higher confidence level, none of the peaks is significant. This is a discouraging result, since the pP phase is clearly visible in the seismic signal, and the SNR is higher than might be expected for many typical earthquakes.

The noisiness in the beam cepstrum and F-statistic is due to high frequency noise in the detrended log-power spectrum. To alleviate the effect of this noise, we have begun experimenting with windowing the low frequency portion of the log spectrum prior to performing cepstrum estimation and peak detection. The dashed box in the flow chart of Figure 1 depicts this modification to the algorithm. The results from processing the data using the modified technique are shown in Figure 4. We windowed the log spectrum of each array component at 2 Hz. The noisiness in both the beam and total cepstrum is significantly reduced, and the F-statistic now clearly detects the correct pP-P delay time with a false alarm rate of .001. Further tests of the method indicate that out-of-band noise in the signal severely degrades the ability to estimate depth phase delay times using straightforward cepstral analysis. Our simple modification to the algorithm improves peak detection capability significantly, using even a simple taper on the detrended log spectrum, and we anticipate that more sophisticated filtering methods will further improve the results.

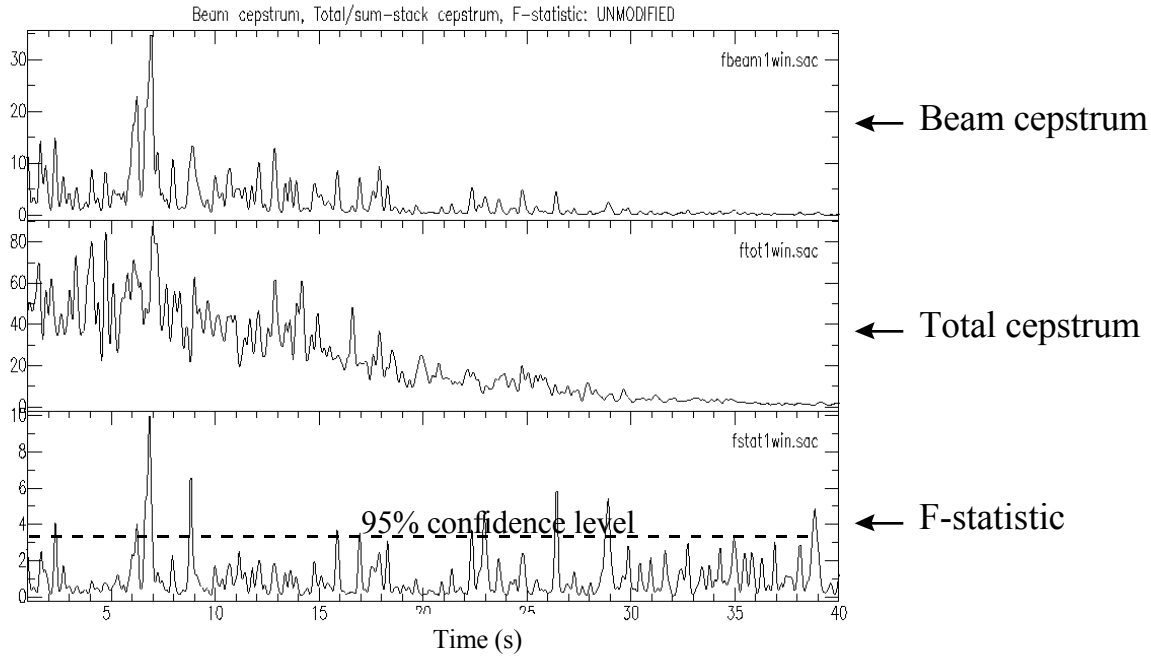

Figure 3. The results from processing the noisy synthetic data using a traditional cepstral analysis with peak detection. The beam cepstrum (top), total (sum-stack) cepstrum (middle) and cepstral F-statistic (bottom) are all noisy, with multiple peaks obscuring the true pP-P delay time (approximately 6.8 seconds).

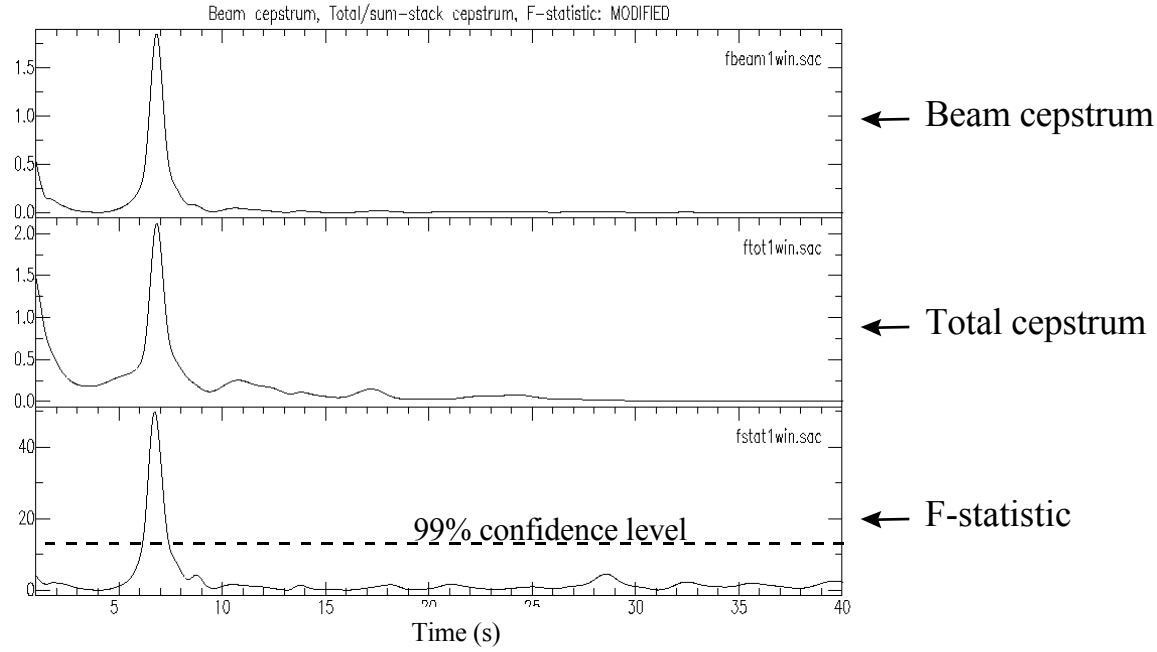

Figure 4. The results from processing the noisy synthetic data using a modified cepstral analysis with peak detection. The modification to the algorithm involves windowing the signal portion of the detrended log spectrum prior to cepstrum estimation and peak detection. The effect of noise on the beam cepstrum, total (sum-stack) cepstrum and cepstral F-statistic is significantly decreased, and the true pP-P delay time is easily detected with high confidence.

We have begun testing our method on a variety of seismic events. For example, we processed the data from an earthquake that appeared in the Reviewed Event Bulletin (REB). This event (EVID 20058461, m_b=4.3) occurred on 19 Aug 1998 in Northern Xinjiang, China. Processing at the International Data Centre (IDC) indicated an event depth of 20.2 km (+-2.2km), and both pP and sP phases were detected at several teleseismic stations. We used a subsample of reporting stations that had higher signal-to-noise ratios, all of which were in the far regional to near-teleseismic range. Table 1 lists the stations used in the cepstral analysis, with the distance from the event, number of windows and length of window for each station provided. For each station, we windowed the detrended log spectra at corner frequencies between 1.3 – 2.0 Hz, depending on where the signal power dropped off. The results are shown in Figure 5 for all five stations used in the analysis. Delay times of approximately 7.2 seconds for pP-P and 10.5 seconds for sP-P results in an estimated depth of 25 km,

Station	Dist	N	Length (s)
NIL	14.37	4	50
BJT	22.42	4	45
PDY	22.54	4	50
FINES	39.04	12	50
ARCES	39.73	15	40

Table 1. Stations and parameters used in cepstral analysis.

which agrees fairly well with the IDC estimate. Note that it is important to see stable peaks in both the beam cepstrum and the F-statistic before interpreting them as actual depth phase delay times (or as any other common signal component). Stable in this case means repeatable over varied processing parameters, such as window length, taper width and passband in the detrended log spectra.

CONCLUSIONS

During the first year of this effort, we have formulated a cepstral focal depth estimation technique that provides a statistical estimate of the significance of peaks in the stacked cepstrum. This significance measure has been missing in previous cepstral estimates of focal depth. The method can be used on both teleseismic and regional data, provided there is sufficient bandwidth and signal strength. We have found that the original formulation of the cepstral F-statistic method is highly sensitive to out-of-band noise, which is amplified in the detrended log spectrum formed prior to cepstrum estimation and peak detection. We have modified the technique by windowing in the detrended log spectrum, and have shown that this modification significantly decreases the influence of noise on the cepstral estimation and peak detection portion of the method.

Future work in the remainder of the effort will include the development of more sophisticated methods to window the signal portion of the detrended log spectra. We also will continue testing different forms of the F-statistic specifically designed for different regimes, such as regional 3-component stations or teleseismic arrays. Application to REB events from regions of monitoring concern is a high priority for future testing, with specific concentration on events with teleseismic depths phases but confusing regional signals. Another important avenue to investigate is the use of preprocessing techniques to further improve the results. These could include well-known signal processing techniques such as band-pass filtering, spectral smoothing and time domain envelope normalization. We are also investigating the use of methods to enhance the depth phase prior to cepstral processing. Our continued major priority during the second year of the effort will be on the operational potential of the method for seismic depth estimation.

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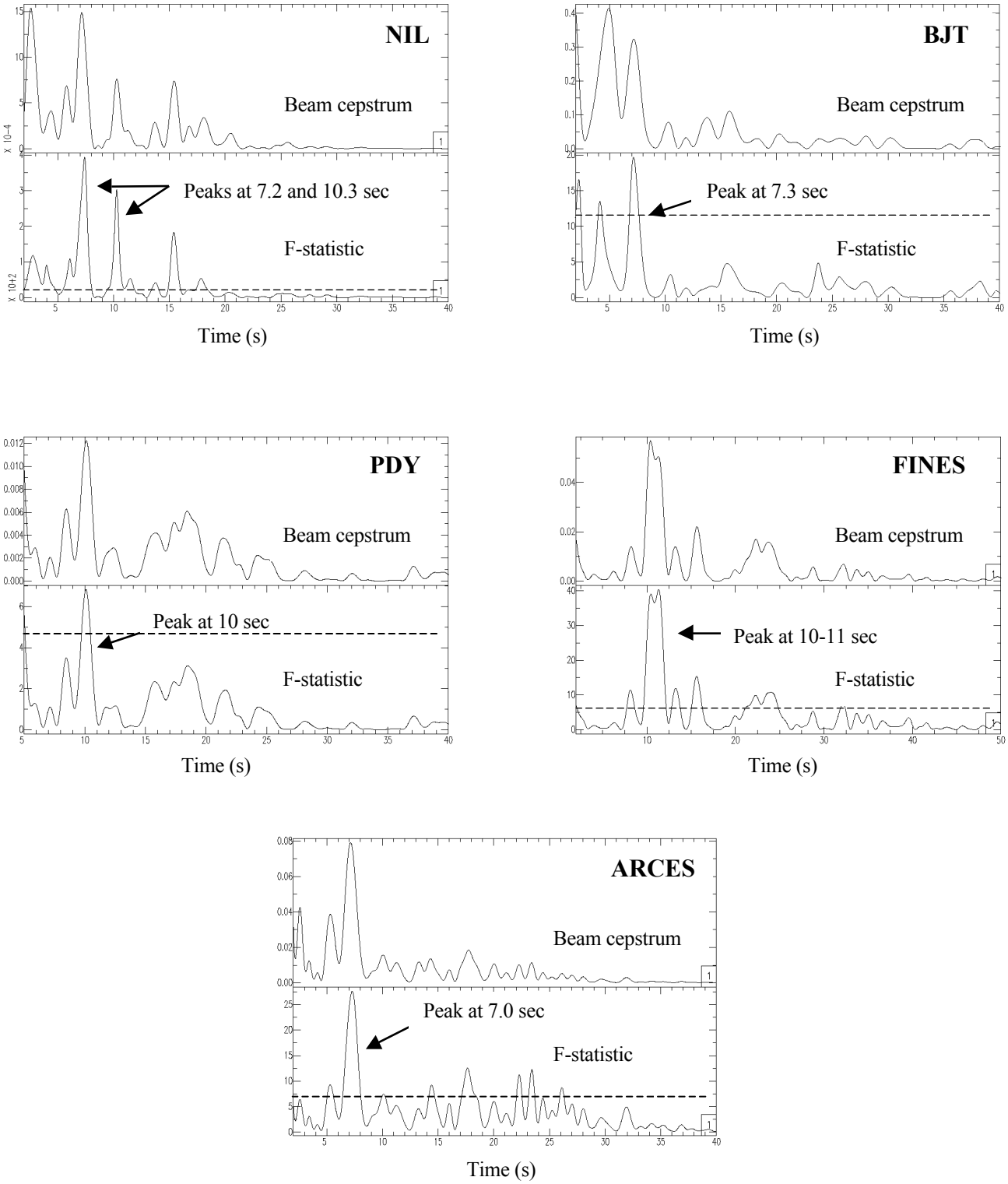

Figure 5. Results from processing data using cepstral analysis from an REB earthquake (evid 20058461, 19 Aug 1998) that took place in Northern Xinjiang, China. Delay times of approximately 7.2 sec for pP-P and 10.5 sec for sP-P result in an estimated depth of 25 km. The dashed lines indicate which peaks are significant at a false alarm rate of .01, except for PDY, which has the line drawn at a false alarm rate of .05.